Wavefunction Dynamics and Application to the Bose Glass Phase

Connor Behan

Department of Physics, Engineering Physics, and Astronomy, Queen's University,
Kingston, ON K7L 3N6, Canada

April 7, 2011

Why Study Low Temperatures?

Large de Broglie wavelength

Quantum effects become important as wavefunctions overlap.

$$\lambda = \frac{h}{\sqrt{3mk_{\mathrm{B}}T}} \tag{1}$$

More interesting free energy

Pure states last longer when the entropy is small.

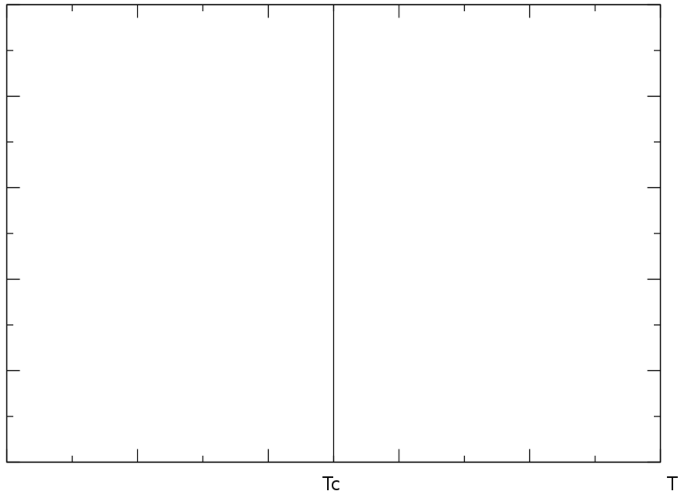
$$\hat{F} = \hat{\mathcal{H}} - T\hat{S} - \mu\hat{N} \tag{2}$$

Quantum statistics

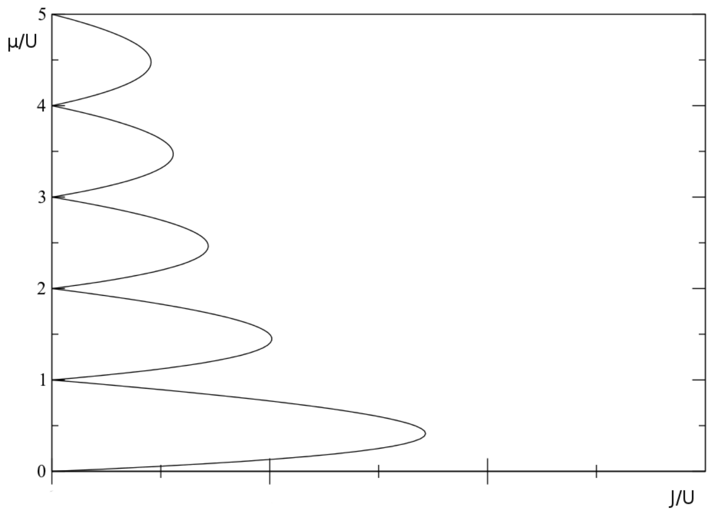
Fermions and bosons begin to show radically different distributions.

$$f_{\mathrm{FD}}(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)}+1}, f_{\mathrm{BE}}(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)}-1} \tag{3}$$

Non-interacting bosons



Interacting bosons on a lattice



Quantum Mechanical Description

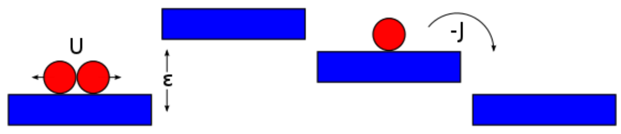
Bose-Hubbard Hamiltonian

$$\hat{\mathcal{H}} = \sum_i \epsilon_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - J \sum_{\langle i,j \rangle \bullet} \hat{a}_i^\dagger \hat{a}_j \tag{4}$$

- Creation and anhilation operators for site i: $\hat{a}_i^\dagger, \hat{a}_i$.
- ϵ_i is the energy well of site i.
- U: how strongly do bosons interact?
- J: how often do bosons tunnel?
- Plot $\langle \hat{n}_i(t) \rangle = \langle \Psi(t) | \hat{a}_i^\dagger \hat{a}_i | \Psi(t) \rangle$ vs t.

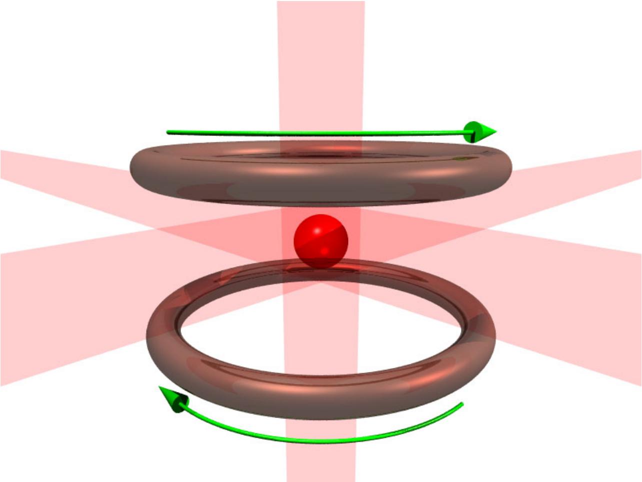
Quantum Mechanical Description

Emergent Phases



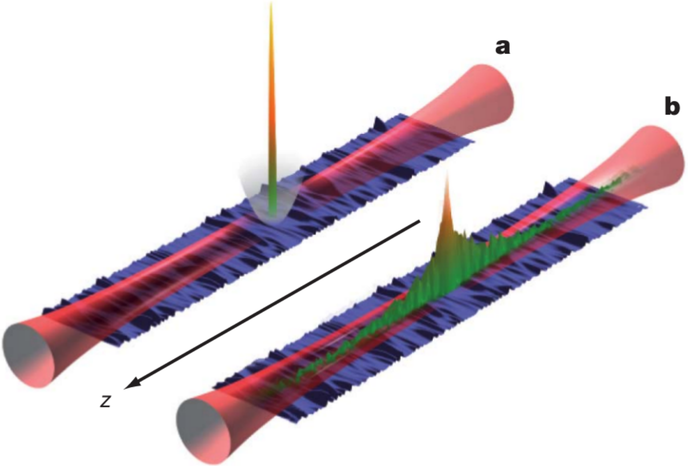
- The Bose glass phase has some isolated regions where bosons are localized [1].
- Of the three phases in the Bose-Hubbard model, it is the only one that requires disorder (not all ϵ_i are the same).
- Mott-insulator: localized bosons on every site.
- Superfluid: probability current flows with zero resistance.

Experimental Situation



Atoms can be confined to the central site of an optical lattice experimentally [2].

Experimental Situation



Bosons diffusing away from the central site in a one-dimensional optical lattice experiment [3].

Some Interesting Results

Discrete Non-linear Schrödinger Equation

$$i\hbar\frac{\mathrm{d}\psi_m(t)}{\mathrm{d}t} = \epsilon_m\psi_m(t) + U|\psi_m(t)|^2\psi_m(t) - J\sum_{\langle j,m\rangle}\psi_j(t) \qquad (5)$$

- Proposed approximation that is much easier to solve [4].
- Plot $|\psi_m(t)|^2$ vs t.

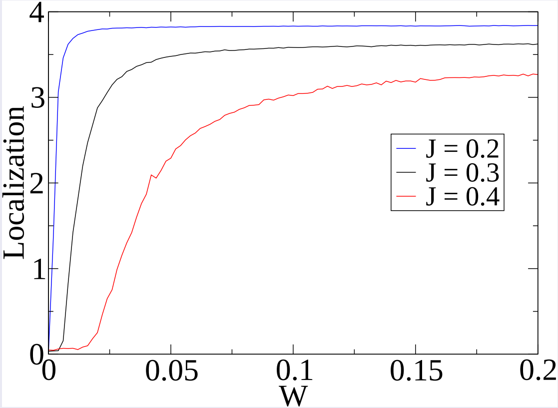
Some Interesting Results

The Effects of Disorder

- The amount of diffusion can depend heavily on whether the bosons start off at the high or low potential.
- Self-trapping has been observed only in the disordered case.
- The system exhibits behaviour similar to that of the Bose glass despite not being in the ground state.
- Qualitative dynamics have been reproduced in experiments [3].

Some Interesting Results

The Effects of Disorder



The amount of localization plotted for an 8 site, 4 boson system with U = 1. W was used to turn up the amount of disorder: $\epsilon = (0, -0.75, -0.34, -0.11, -0.36, -0.69, -0.23, -0.3)W$.

Conclusion

- Many-body physics on small systems was effectively simulated.
- The dynamics of the wavefunctions were plausible.
- The discrete non-linear Schrödinger equation was found to be inadequate at describing this diffusion.
- The mean-field dynamical system might yield a better approximation.

References

V. I. Yukalov (2000)
Cold Bosons in Optical Lattices.
Laser Physics 19(1), 1 – 110.

V. Vuletic; T. W. Hänsch; C. Zimmermann (1996)
Steep Magnetic Trap for Neutral Atoms.
Europhysics Letters 36.

J. Billy; V. Josse; Z. Zuo; A. Bernard; B. Hambrecht; P. Lugan; D. Clement; L. Sanchez-Palencia; P. Bouyer; A. Aspect (2008)
Direct observation of Anderson localization of matter waves in a controlled disorder.
Nature 453, 891 – 894.

S. Flach; D. O. Krimer; C. Skokos (2009)
Universal Spreading of Wave Packets in Disordered Nonlinear Systems.
Physical Review Letters 102(2).